MATH 162A Review: Abstract Vector Space

Facts to Know:

V: $\{v_1 \cdots v_k\}$ is called a $\boxed{\text{basis}}$

① $\{v_1 \cdots v_k\}$ are L.I.

② span $\{v_1 \cdots v_k\} = V$.

The definition of vector space; linear dependence and independence; basis and dimension.

dimension $= k$.

$(V, +, \cdot)$

① $u+v=v+u$

② $(u+v)+w = u+(v+w)$

③ $0+u = u+0 = u$

④ $(rs)\cdot u = r\cdot(s\cdot u)$

⑤ $(r+s)u = ru+su$

⑥ $r(u+v) = ru+rv$

⑦ $0\cdot u = 0$

⑧ $1\cdot u = u$

$v_1 \cdots v_k$ are called L.I. if

The equation

$c_1 v_1 + \cdots + c_k v_k = 0$

has zero solution $(c_1 \cdots c_k) = (0 \cdots 0)$

Examples:

1. Space of polynomials of degree no more than n is a vector space

$\underline{p(t)+q(t)}$ are polynomials of $\underline{\text{degree} \le n}$

$\underline{r\cdot p(t)}$

2. Space of real-valued functions

$(f+g)(x) = f(x)+g(x)$

$(r\cdot f)(x) = r f(x)$

3. How to define linear independence?

$p(t) = 0.$ $\quad p(t) = a_0 + a_1 t + \cdots + a_n t^n$

$p(t) = 0 \iff a_0 = 0,\ a_1 = 0, \cdots a_n = 0$

$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ $\quad x = 0 \iff x_1 = 0 \cdots x_n = 0.$

$\{x^2, x\}$

$\boxed{c_1 p_1 + \cdots + c_k p_k = 0}$

$c_1 x^2 + c_2 x = 0$

$c_1 = 0,\ c_2 = 0$